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# Exact Linearizations of Input Output Systems

L.R. Hunt* and M. Luksic*
Department of Mathematics
Texas Tech University
Lubbock, Texas 79409
U.S.A

Renjeng Su+
Department of Electrical Engineering
Texas Tech University
Lubbock, Texas 79409
U.S.A

## Abstract

The problem of transforming nonlinear systems to linear systems is receiving much attention in the literature. In this paper we present necessary and sufficient conditions that a nonlinear control system with output be equivalent to a linear control system with linear output (which is controllable and observable). The conditions depend on certain Lie derivatives of the output and can be verified in a finite number of steps. For simplicity we consider only the single input, single output case.

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## 1. Introduction

The main problem of interest is to determine if a given nonlinear system with output is equivalent to a controllable and observable linear system. For this paper we restrict to a real-analytic single input, single output nonlinear system

$$
\begin{aligned}
\dot{x} &= f(x) + g(x)u \\
y &= h(x)
\end{aligned}
\tag{1}
$$

with $f(0) = 0$. The first equation in (1) represents the dynamics, the second the output. By equivalence we mean there exists a nonsingular smooth state space coordinate change on $\mathbb{R}^n$ near 0 so that (1) becomes

$$
\begin{aligned}
\dot{w} &= Aw + bv \\
z &= Cw
\end{aligned}
\tag{2}
$$

a controllable and observable system. For this "state space equivalence" we take $v = u$. The conditions we derive involve a finite number of Lie derivatives of the output function $h(x)$, in (1), and these appear as kernels in the (formal) Volterra series of (1).

If we ignore the outputs in systems (1) and (2), the state space equivalence problem is solved in [1] and [2]. Isidori [3] also presents conditions under which the nonlinear system with output exhibits linear input-output behavior.

Another type of equivalence that has received much attention in the literature is the feedback equivalence (state space coordinate changes, nonlinear feedback, and input space coordinate changes). The feedback equivalence problem for nonlinear systems and linear systems is now well understood (see [4] and [5]) when outputs are not considered. If outputs are added, then the feedback equivalence problem, where we wish to move (1) to a system which has linear input-output behavior in some specified sense, is examined in [3],[6],[7] and [8].

In section 2 of this paper we introduce needed definitions and notation. The third section contains our main results. Here we present sufficient conditions involving Lie derivatives of the output for system (1) so that (near the origin in $\mathbb{R}^n$)

(i) the system $\dot{x} = f(x) + g(x)u$ is feedback equivalent to $\dot{w} = Aw + bv$,

(ii) the system $\dot{x} = f(x) + g(x)u$ is state equivalent to $\dot{w} = Aw + bv$,

(iii) the input-output system (1) is state equivalent to the input-output system (2). The sufficient conditions here are also necessary.

Multi input, multi output versions of these results will appear elsewhere.

For interesting results on the equivalence of nonlinear systems (without inputs) to linear systems having output injection (also without inputs) we refer to [9], [10], and [11]. In [11], the problem where inputs are added is also considered, but the results are more concerned with observer design than with equivalence conditions. The observer design technique with inputs is related to the design technique without inputs.

## 2. Definitions

If f and g are $C^\infty$ vector fields on $\mathbb{R}^n$, then the Lie bracket is

$$[f,g] = \frac{\partial f}{\partial x} g - \frac{\partial g}{\partial x} f$$

(this is the negative of the usual definition), where $\frac{\partial f}{\partial x}$ and $\frac{\partial g}{\partial x}$ are Jacobian matrices. Successive Lie brackets are defined by

$$\begin{aligned}
(ad^0 f,g) &= g \\
(ad^1 f,g) &= [f,g] \\
(ad^2 f,g) &= [f,[f,g]] \\
&\vdots \\
(ad^k f,g) &= [f, (ad^{k-1} f,g)].
\end{aligned}$$

Given a $C^\infty$ function h and a $C^\infty$ vector field f, the Lie derivative of h with respect to f is

$$L_f h = \langle dh, f \rangle,$$

where $\langle \cdot, \cdot \rangle$ denotes the duality between one forms and vector fields. Then we let

$$\begin{aligned}
L_f^0 h &= h \\
L_f^1 h &= L_f h \\
L_f^2 h &= L_f(L_f h) = L_f L_f h \\
&\vdots \\
L_f^k h &= L_f(L_f^{k-1} h) = L_f L_f^{k-1} h.
\end{aligned}$$

Other Lie derivatives like $L_g L_f^k h$ can be defined.

Starting with system (1) and moving to system (2) through equivalence, one assumption which we have indicated is that system (2) be controllable and observable. To guarantee this, for the remainder of the paper both of the sets $\{g, [f,g], \ldots, (ad^{n-1} f, g)\}$ and $\{dh, dL_f h, \ldots, dL_f^{h-1} h\}$ are assumed to span $\mathbb{R}^n$ near the origin.

We also make use of the Lie derivative of a one form $\omega$ with respect to a vector field f

$$L_f(\omega) = \left(\frac{\partial w}{\partial x}^* f\right)^* + \omega \frac{\partial f}{\partial x} ,$$

with * denoting transpose.

The three types of Lie derivatives are related by the formula

$$L_f\langle\omega, g\rangle = \langle L_f(\omega), g\rangle - \langle\omega, [f,g]\rangle . \tag{3}$$

Let h be a $C^\infty$ function and g a $C^\infty$ vector field in some neighborhood U of the origin in $\mathbb{R}^n$. If dh does not vanish at 0, there is an open set, also called U, containing 0 so that U is foliated by the (n-1) dimensional level sets of h. Starting at all initial points in the level set $S_0$ of h through 0, we assume that U consists of all solutions of $\dot{x}(t) = g(x(t))$ for t in some interval $(-t_0, t_0)$, $t_0 > 0$. For $t \in (-t_0, t_0)$, let $S_t$ be the set of all points x(t), where x(t) solves $\dot{x}(t) = g(x(t))$ and $x(0) \in S_0$.

We remark that U can be reduced, if necessary, in the proof of the following result.

Lemma 2.1. If $L_g h$ is a constant in $U$, then for all $t \varepsilon (-t_0, t_0)$, $S_t$ is a level set of $h$.

Proof. Since $g$ is nonvanishing on $U$, coordinate changes exist so that

$$g = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix}$$

on $U$. If $L_g h = 0$ on $U$, then $h$ is independent of $x_n$ since $\langle dh, g \rangle = \frac{\partial h}{\partial x_n} = 0$. Hence the integral curve of $\dot{x} = g(x(t))$ through any point in $S_t$ is actually contained in $S_t$.

Suppose $L_g h = C_0$, $C_0 \neq 0$, on $U$. Then $\frac{\partial h}{\partial x_n} = C_0$ on $U$ and $h$ is linear in $x_n$. The level sets of $h$ are given by $x_n - \tilde{h}(x_1, x_2, \ldots, x_{n-1}) = d$, where $d$ is a constant for each level set ($d = 0$ for the level set through 0) and $\tilde{h}$ has the obvious definition.

Since the solution of $\dot{x}(t) = g(x(t))$ starting at any point in $S_0$ is $x_1$ = constant, $x_2$ = constant, ..., $x_{h-1}$ = constant, $x_n = t$, it is obvious that the flow maps the level sets as required. $\square$

The proof of the following lemma involves easy computations and is left to the reader.

Lemma 2.2. The condition that $L_g h$ is a constant on $U$ is invariant under nonsingular coordinate changes on $\mathbb{R}^n$.

In addition to the assumption in this lemma, suppose also that we are in a coordinate system so that $g$ is a constant vector field. We differentiate $L_g h$ with respect to $x_1$, with respect to $x_2, \ldots,$ and with respect to $x_n$ to find

$$Hg = 0,$$

where H is the $n \times n$ Hessian matrix

$$\begin{bmatrix} \dfrac{\partial^2 h}{\partial x_1^2} & \cdots & \dfrac{\partial^2 h}{\partial x_1 \partial x_n} \\ \vdots & & \vdots \\ \dfrac{\partial^2 h}{\partial x_n \partial x_1} & \cdots & \dfrac{\partial^2 h}{\partial x_n^2} \end{bmatrix}$$

Since g is nonvanishing, we have the determinant of H is identically zero. This gives us the homogeneous real Monge-Ampere equations [12]

$$\det H = 0.$$

Moreover, g is contained in the Monge-Ampere foliation. If

$$g = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},$$

then h must be linear in the $x_n$ variable, as indicated in the proof of Lemma 2.1.

We present definitions concerning equivalence of two systems. By <u>state equivalence</u> we mean there exists a nonsingular coordinate change on $\mathbb{R}^n$ taking one system to the other. By <u>feedback equivalence</u> we mean there exists a nonsingular transformation involving state space coordinate changes, feedback, and coordinate changes on the input space. As shown in [5] this can be viewed as a map from $\mathbb{R}^{n+1}$ to $\mathbb{R}^{n+1}$ (if systems (1) and (2) are considered, (x,u) space goes to (w,u) space).

## 3. Main Results

We examine conditions under which we can move from system (1) to system (2). A simple example illustrates our approach to this problem.

Example 3.1. Consider the nonlinear system

$$(4) \qquad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 - x_3^2 + 2(x_2 - x_3^2)x_3 \\ x_3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2x_3 \\ 1 \end{bmatrix} u = f(x) + g(x)u$$

$$y = h(x) = x_1 - x_2^2 + 2x_2x_3^2 - x_3^4 + x_2 - x_3^2$$

on a neighborhood of 0 in $\mathbb{R}^3$. The state space coordinate change

$$(5) \qquad \begin{aligned} w_1 &= x_1 - (x_2 - x_3^2)^2 \\ w_2 &= x_2 - x_3^2 \\ w_3 &= x_3 \end{aligned}$$

takes (4) to (with v = u)

$$(6) \qquad \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \dot{w}_3 \end{bmatrix} = \begin{bmatrix} w_2 \\ w_3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v$$

$$z = w_1 + w_2 ,$$

a linear system with linear output. Thus we have that (4) is state equivalent to (6).

An easy calculation shows that each of the sets $\{g, [f,g], (ad^2 f,g)\}$ and $\{dh, dL_f h, dL_f^2 h\}$ for system (4) are linearly independent. This is equivalent to the fact that the linear system (6) is controllable and observable.

Next we examine computations involving the output function $y = h(x)$ in (4). It can be shown that the Lie derivatives $L_g L_f^k h(x)$, $k = 0,1,2,3,4,5$, are all constants for x near 0. We prove later (for the general case) that this implies

$$[(ad^s f,g),\ (ad^t f,g)] = 0$$

for $0 \leq s,\ t \leq 3$, and the dynamics in (4) (without output) are equivalent to the dynamics in (6) (see [1] and [2]). The coordinate changes to move from (4) to (6) are exactly those given in (5), under which the output in system (6) is linear.

Given system (1) our main concerns are to determine sufficient conditions so that the results (i), (ii), and (iii) as stated in the introduction hold.

Theorem 3.1. (i) If there exist constants $C_k$ so that $L_g L_f^k h = C_k$, $k = 0,1,\ldots,2n-3$ on a neighborhood of the origin in $\mathbb{R}^n$, then the dynamics $\dot{x} = f(x) + g(x)u$ in the nonlinear system (1) are feedback equivalent to the dynamics $\dot{w} = Aw + bv$ in (2).

(ii) If there exist constants $C_k$ so that $L_g L_f^k h = C_k$, $k = 0, 1,\ldots, 2n-1$ on a neighborhood of the origin in $\mathbb{R}^n$, then

$\dot{x} = f(x) + g(x)u$ is state equivalent to $\dot{w} = Aw + bv$.

(iii) If there exist constants $C_k$ so that $L_g L_f^k h = C_k$, $k = 0, 1, \ldots, 2n-1$ on a neighborhood of the origin in $\mathbb{R}^n$, then the input-output nonlinear system (1) is state equivalent to the input-output linear system (2). Moreover, the converse is also true.

We remark that (iii) obviously implies (ii), but it is natural that (ii) be proved before (iii).

Proof. Since the conditions $L_g L_f^k h = C_k$ are invariant under state space coordinate changes by Lemma 2.2, we define new coordinates

$$
\begin{aligned}
T_1 &= h(x) \\
T_2 &= L_f h(x) \\
&\vdots \\
T_n &= L_f^{n-1} h(x).
\end{aligned}
\tag{7}
$$

In these T coordinates, which for simplicity we now call x coordinates, equations (1) become

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ x_n \\ f_n(x_1, x_2, \ldots, x_n) \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{n-1} \\ g_n \end{bmatrix} u
\tag{8}
$$

$$y = x_1.$$

Now

$$L_g h = C_0 \text{ implies } g_1 = C_0$$

$$L_g L_f h = C_1 \text{ implies } g_2 = C_1$$

$$\vdots$$

$$L_g L_f^{n-1} h = C_{n-1} \text{ implies } g_n = C_{n-1}.$$

We also have

$$L_g L_f^n h = C_n = \sum_{i=1}^{n} \frac{\partial f_n}{\partial x_i} C_{i-1}$$

and

$$[f,g] = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_{n-1} \\ \sum_{i=1}^{n} \frac{\partial f_n}{\partial x_i} C_{i-1} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_{n-1} \\ C_n \end{bmatrix}.$$

Applying the definitions of Lie derivatives and equation (3) we find

$$\begin{aligned} L_{[f,g]} L_f^n h &= \langle dL_f^n h, [f,g] \rangle \\ &= \langle L_f (dL_f^n h), g \rangle - L_f \langle dL_f^n h, g \rangle \\ &= \langle dL_f L_f^n h, g \rangle - L_f \langle dL_f^n h, g \rangle \\ &= \langle dL_f^{n+1} h, g \rangle - L_f \langle dL_f^n h, g \rangle \\ &= L_g L_f^{n+1} h - L_f L_g L_f^n h \end{aligned}$$

But

$$L_g L_f^n h = C_n$$

yields

$$L_{[f,g]} L_f^n h = L_g L_f^{n+1} h = C_{n+1}.$$

Now

$$L_{[f,g]} L_f^n h = \sum_{i=1}^{n} \frac{\partial f_n}{\partial x_i} C_i = C_{n+1}$$

and

$$(ad^2 f,g) = \begin{bmatrix} C_2 \\ C_3 \\ \cdot \\ \cdot \\ \cdot \\ C_n \\ C_{n+1} \end{bmatrix}.$$

Similarly,

$$L_{(ad^2 f,g)} L_f^n h = L_{[f,g]} L_f^{n+1} h = L_g L_f^{n+2} h = C_{n+2}$$

and

$$\sum_{i=1}^{n} \frac{\partial f_n}{\partial x_i} C_{i+1} = C_{n+2}.$$

Hence

$$(ad^3 f,g) = \begin{bmatrix} C_3 \\ C_4 \\ \cdot \\ \cdot \\ \cdot \\ C_{n+1} \\ C_{n+2} \end{bmatrix}.$$

Continuing in this way and assuming the hypothesis in (i) holds we have

$$g = \begin{bmatrix} c_0 \\ c_1 \\ \cdot \\ \cdot \\ \cdot \\ c_{n-2} \\ c_{n-1} \end{bmatrix}, \ [f,g] = \begin{bmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ \cdot \\ c_{n-1} \\ c_n \end{bmatrix}, \ \ldots \ (ad^{n-2}f,g) = \begin{bmatrix} c_{n-2} \\ c_{n-1} \\ \cdot \\ \cdot \\ \cdot \\ c_{2n-4} \\ c_{2n-3} \end{bmatrix},$$

These vector fields form an involutive set and the conclusion in (i) is valid by results on feedback equivalence in [5].

If $L_g L_f^{2n-2} = c_{2n-2}$ and $L_g L_f^{2n-1} = c_{2n-1}$ also hold then

$$(ad^{n-1}f,g) = \begin{bmatrix} c_{n-1} \\ c_n \\ \cdot \\ \cdot \\ \cdot \\ c_{2n-3} \\ c_{2n-2} \end{bmatrix} \text{ and } (ad^n f,g) = \begin{bmatrix} c_n \\ c_{n+1} \\ \cdot \\ \cdot \\ \cdot \\ c_{2n-2} \\ c_{2n-1} \end{bmatrix}.$$

Therefore

$$[(ad^s f,g),(ad^t f,g)] = 0$$

for all $0 \leq s,t \leq n$, and (ii) is proved by results in [2].

At this point we have shown that

$$\begin{aligned} &\langle df_n, g\rangle = c_n \\ (9) \qquad &\langle df_n, [f,g]\rangle = c_{n+1} \\ &\quad\vdots \\ &\langle df_n, (ad^{n-1}f,g)\rangle = c_{2n-1}. \end{aligned}$$

Since $g, [f,g], \ldots,$ and $(ad^{n-1}f,g)$ are constant vector fields, differentiation of each equation in (9) with respect to $x_1$, with respect to $x_2, \ldots,$ and with respect to $x_n$ yields

$$\left[\frac{\partial^2 f_n}{\partial x_i \partial x_j}\right] \ell = 0.$$

Here $\left[\frac{\partial^2 f_n}{\partial x_i \partial x_j}\right]$ denotes the Hessian matrix of $f_n$ and $\ell$ is any vector field in $\{g, [f,g], \ldots, (ad^{n-1}f,g)\}$. Because this set of vectors is linearly independent set we must have that $f_n$ is linear in $x_1, x_2, \ldots, x_n$ (recall $f(0) = 0$). Alternatively, we have a full Monge-Ampere foliation implying $f_n$ is linear. Hence (1) is then a linear system with linear output.

The necessity in (iii) is trivial since the conditions $L_g L_f^k h = C_k$, $k = 0, 1, \ldots, 2n-1$ hold for a linear system and are invariant under coordinate changes on $\mathbb{R}^n$. $\square$

In [3] Isidori introduces the Lie derivatives of an input-output nonlinear system (1) in (formal) Volterra series.

$$(10) \qquad y(t) = w^{(0)}(t,x) + \int_0^t w^{(1)}(t,\tau_1,x)u_i(\tau_1) + \ldots,$$

where

$$w^{(0)}(t,x) = \sum_{k=0}^{\infty} L_f^k h(x) \frac{t^k}{k!}$$

$$w^{(1)}(t,\tau_1,x) = \sum_{k_1,k_2=0}^{\infty} L_f^{k_2} L_g L_f^{k_1} h(x) \frac{(t-\tau_1)^{k_1}}{k_1!} \frac{\tau_1^{k_2}}{k_2!} .$$

He remarks that if the $L_g L_f^k h(x)$ are independent of x for all $k \geq 0$, then the input-dependent part of the response of the nonlinear system (1) is linear in the input. Putting conditions on (1) so that the resulting linear system (2) is controllable and observable, we have shown that only the first 2n of $L_g L_f^k h$ need to be considered. In addition, the Volterra series collapses to the variation of constants formula in our case.

Following Isidori [3], an interesting problem for (1) is to determine conditions under which there are functions $\alpha$ and $\beta$ so that $L_{\beta g} L^k_{(f+g\alpha)} h(x)$ are independent of x for $k = 0,1,\ldots, 2n-1$. For the dynamics in (1) to be feedback equivalent to the dynamics in (2) it is necessary that the set $\{g,[f,g],(ad^2 f,g),\ldots,(ad^{n-2} f,g)\}$ be involutive (see [4] or [5]). One way in which the transformation from a nonlinear system to a linear system is accomplished is by finding $\alpha$ and $\beta$, and then following with state space coordinate changes [13]. Then all we have to ask in order for the input-output nonlinear system (1) to be feedback equivalent to the input-output linear system (2) is a simple question. Do the $\alpha$ and $\beta$ satisfy the conditions that $L_{\beta g} L^k_{f+\alpha g} h(x)$, $k=0,1,\ldots,2n-1$ are independent of x? Of course this requires finding $\alpha$ and $\beta$, which can be quite difficult (if not impossible) in many cases. Moreover, it is desirable to determine necessary and sufficient conditions depending only on f, g and h and not on $\alpha$ and $\beta$. However, this may not be possible since the conditions $L_g L_f^k h(x)$ = constant, $k \geq 1$, are not invariant under feedback.

George Meyer's [14] highly successful applications of nonlinear transformation theory to automatic control of aircraft can be viewed in light of this paper. Our discussion is restricted to a single-input, single-output case. Meyer's [14] mathematical model is block triangular, i.e.

$$
\begin{aligned}
\dot{x}_1 &= f_1(x_1,x_2) \\
\dot{x}_2 &= f_2(x_1,x_2,x_3) \\
&\vdots \\
\dot{x}_{n-1} &= f_{n-1}(x_1,x_2,\ldots,x_n) \\
\dot{x}_n &= f_n(x_1,x_2,\ldots,x_n) + u
\end{aligned}
\tag{11}
$$

with output $y = x_1$.

Using the transformation theory in [14] we obtain the linear system

$$
\begin{aligned}
\dot{w}_1 &= w_2 \\
\dot{w}_2 &= w_3 \\
&\vdots \\
\dot{w}_n &= v
\end{aligned}
\tag{12}
$$

with output $z = w_1 = x_1$, a linear system with linear output. We can easily calculate the $\alpha$ and $\beta$ so the conditions mentioned in the preceding paragraph are satisfied. We remark that system (12) has no zeros, and is thus a special case. The general theory of this page allows for zeros, as illustrated by example (4).

The multi-input, multi-output theory is a part of the thesis of Mladen Lukšic.

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